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tained in its subgroup of order p^{a-1}. That is, $\phi(p^a)=p^a-p^{a-1}=p^a(1-1/p)$. If $m=p_1^{a_1}p_2^{a_2}\ldots\ldots p_\rho^{a_\rho}$, it follows from this and the preceding formula that

$$\phi(m)=\phi(p_1^{a_1})\phi(p_2^{a_2})\ldots\ldots\phi(p_\rho^{a_\rho})$$

$$=p_1^{a_1}p_2^{a_2}\ldots\ldots p_\rho^{a_\rho}(1-\frac{1}{p_1})(1-\frac{1}{p_2})\ldots\ldots(1-\frac{1}{p_\rho})$$

$$=m(1-\frac{1}{p_1})(1-\frac{1}{p_2})\ldots\ldots(1-\frac{1}{p_\rho}).$$

It may be observed that the formula $\phi(p^a)=p^{a-1}(p-1)$ may be regarded as a special case of the formula $\phi(m^r)=m^{r-1}\phi(m)$, since $\phi(p)$ is evidently equal to $p-1$.

AN APPLICATION OF STIRLING'S INTERPOLATION FORMULA

By FRANK ELMORE ROSS.

Let write the values of sin$x$ for $x=0$, π, etc., and form the series of differences, as in the ordinary process of interpolation. The result is as follows, Δ_n denoting the nth difference:

x	sin$x$	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	....
0	0						
		+1					
$\frac{1}{2}\pi$	+1		−2				
		−1		+2			
π	0		0		0		
		−1		+2		−4	
$3\pi/2$	−1		+2		−4		....
		+1		−2		+4	
0	0		0		0		....
		+1		−2		+4	
$\frac{1}{2}\pi$	+1		−2		+4		
		−1		+2			
π	0		0				
		−1					
$3\pi/2$	−1						

This scheme extends infinitely in both directions, the difference Δ_n forming a divergent series. It is found however that a convergent series for the function thus tabulated can be constructed from the above series of differences, by making use of Stirling's interpolation formula.

$$f(a+nw)=f(a)+n\Delta_1+\frac{n^2}{2!}\Delta_2+\frac{n(n^2-1)}{3!}\Delta_3+\frac{n^2(n^1-1)}{4!}\Delta_4$$

$$+\frac{n(n^2-1)(n^2-2^2)}{5!}\Delta_5 \;\cdots\cdots\cdots\; (1).$$

In this formula Δ_n stands for the differences which lie in the same horizontal line $f(a)$, in the case of the even differences, and in the case of the odd differences, for the mean of those differences which lie just above and below this line. If we apply (1) to the above scheme, we get at once

$$\sin x=x\left[1-\frac{2(x^2-1)}{3!}+\frac{2^2(x^2-1)(x^2-2^2)}{5!}-\frac{2^3(x^2-1)(x^2-2^2)(x^2-3^2)}{7!}+\cdots\cdots\right.$$

$$\left.\cdots\cdots\cdots+\frac{(-2)^n(x^2-1)\cdots\cdots\cdots(x^2-n^2)}{(2n+1)!}-\cdots\cdots\cdots\right]\cdots\cdots\cdots (2),$$

in which x is expressed in terms of π. This series is convergent for all values of x, since the ratio of two successive terms

$$\frac{x^2-n^2}{n(2n+1)}\doteq\tfrac{1}{2}\quad(n\doteq\infty).$$

While this series is not of any practical use for computing this function, it is nevertheless believed to be new. From it we can readily obtain a simple series for π, as follows:

Since $\frac{\sin x}{x}\doteq\frac{1}{2}\pi$, $(x\doteq0)$, we get by putting $x=0$ in (2),

$$\tfrac{1}{2}\pi=1+\frac{2}{3!}+\frac{2^2 2^2}{5!}+\frac{2^3 2^2 2^3}{7!}+\cdots\cdots\cdots+\frac{2^n 2^2 3^2\cdots\cdots\cdots n^2}{(2n+1)!}\cdots\cdots\cdots, \qquad \cdots\cdots\cdots(3)$$

which is convergent, the ratio of convergence being the same as in (2). This series is also believed to be new. It is found to be quite convenient for arithmetical computation. By means of it the value of π correct to eight decimal places was computed in half an hour. There are however better series for this purpose.